Section 5.2 Integration by "u" substitution

These are the only rules for integration that we use in our class. We were able to rewrite all the problems in section 5.1 to fit one or more of the rules using basic Algebra.

The problems in section 5.2 will also need to be rewritten so that we can apply the integration rules we learned in section 5.1. The rewriting is a little trickier. The technique we will use to rewrite each problem so that it we can integrate is called u $- \mathit{substitution}$.

Basic Integration Rules

Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ n \neq -1$

Integral of a constant Rule: $\int a dx \ = \ ax + C \quad (a\ is\ any\ real\ number)$

"ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

"e" Rule $\int e^x dx \ = e^x + C$

$C\ represents\ any\ real\ number$

Properties of integrals

$\int \big(f(x) + g(x)\big)dx = \int f(x)dx \quad + \int g(x)dx$

$\int \big(f(x) - g(x)\big)dx = \int f(x)dx \quad - \int g(x)dx$

$\int af(x)dx = a \int f(x)dx$

Example: Rewrite a problem that has a parenthesis with an exponent using $u - substitution$, then integrate to find the answer.

$\int 10x(5x^2 - 4)^5 dx$

Rewrite the problem so that the parenthesis is first:

$= \int (5x^2 - 4)^5 10x dx$

Next: $let\ u = inside\ of\ the\ parenthesis$

$let\ u = 5x^2 - 4$

Rewrite the problem so that the “parenthesis is changed to an "u"

$= \int u^5 10x dx$

Next find $\frac{du}{dx}$

$u = 5x^2 - 4$

$\frac{d}{dx} u = \frac{d}{dx} 5x^2 - \frac{d}{dx} 4$

$\frac{du}{dx} = 10x$

Multiply by dx to clear the fraction.

$dx \frac{du}{dx} = 10x dx$

$du = 10x dx$

Next replace $10xdx\ with\ du$

$= \int u^5 10xdx\ = \int u^5 du$

Next integrate: use Power Rule

$= \frac{1}{6}u^6 + C$

Last change $u\ to\ 5x^2 - 4\ to\ get\ the\ answer$

$Answer$: $\frac{1}{6}(5x^2 - 4)^6 + C$

Example: Rewrite a problem that has a parenthesis with an exponent using $u - substitution$, then integrate to find the answer.

$\int 20x(5x^2 - 4)^5 dx$

Rewrite the problem so that the parenthesis is first:

$= \int (5x^2 - 4)^5 20xdx$

Next: $let\ u = inside\ of\ the\ parenthesis$

$let\ u = 5x^2 - 4$

Rewrite the problem so that the “parenthesis is changed to an "u"

$= \int u^5 20xdx$

Next find $\frac{du}{dx}$

$u = 5x^2 - 4$

$\frac{d}{dx}u = \frac{d}{dx}5x^2 - \frac{d}{dx}4$

$\frac{du}{dx} = 10x$

Multiply by dx to clear the fraction.

$dx\frac{du}{dx} = 10xdx$

$du = 10xdx$

This is not good enough. I need to replace $20xdx$.

Multiply by 2.

$2du = 2 * 10xdx$

$2du = 20xdx$

Next replace $20xdx$ $with\ 2du$

$= \int u^5 20xdx = \int u^5 2du = \int 2u^5 du$

Rewrite using: $\int af(x)dx = a \int f(x)dx$

$= 2 \int u^5 du$

Next integrate: use Power Rule:

$= 2 * \frac{1}{6} u^6 + C$

$= \frac{1}{3} u^6 + C$

Last change $u\ to\ 5x^2 - 4\ to\ get\ the\ answer$

$Answer$: $\frac{1}{3}(5x^2 - 4)^6 + C$

Example: Rewrite a problem that has an "e"using $u - substitution$, then integrate to find the answer.

Find: $\int 12x^2 e^{4x^3} dx$

Rewrite the problem with the e first.

$= \int e^{4x^3} 12x^2 dx$

Let $u = exponent\ of\ the\ e$

Let $u = 4x^3$

Rewrite the problem with $u\ in\ the\ exponent\ of\ the\ e$

$= \int e^u 12x^2 dx$

Next find $\frac{du}{dx}$

$u = 4x^3$

$\frac{d}{dx} u = \frac{d}{dx} 4x^3$

$\frac{du}{dx} = 12x^2$

Multiply by dx to clear the fraction.

$dx \frac{du}{dx} = dx * 12x^2$

$du = 12x^2 dx$

Next replace $12x^2\ dx\ with\ du$

$= \int e^u 12x^2 dx$

$= \int e^u du$

Integrate: Use "e" Rule $\int e^x dx \ = e^x + C$

= $e^u + C$

Replace the u with $4x^3 and\ write\ answer$

Answer: $e^{4x^3} + C$

Example: Rewrite a problem that has an "e" using $u - substitution$, then integrate to find the answer.

Find: $\int 2x^2 e^{4x^3} dx$

Rewrite the problem with the e first.

$= \int e^{4x^3} 2x^2 dx$

Let $u = exponent\ of\ the\ e$

Let $u = 4x^3$

Rewrite the problem with $u\ in\ the\ exponent\ of\ the\ e$

$= \int e^u 2x^2 dx$

Next find $\frac{du}{dx}$

$u = 4x^3$

$\frac{d}{dx} u = \frac{d}{dx} 4x^3$

$\frac{du}{dx} = 12x^2$

Multiply by dx to clear the fraction.

$dx \frac{du}{dx} = dx * 12x^2$

$du = 12x^2 dx$

Multiply by $\frac{1}{6}$ *to get a match of what is left unchanged inside the integral*

$du = 12x^2dx$

$\frac{1}{6}du = \frac{1}{6} * 12x^2dx$

$\frac{1}{6}du = 2x^2dx$

Replace the $2x^2dx$ *with* $\frac{1}{6}du$

$= \int e^u 2x^2dx$

$= \int e^u \frac{1}{6}du$ = $\int \frac{1}{6}e^u du$

Rewrite: using $\int af(x)dx = a\int f(x)dx$

$= \frac{1}{6}\int e^u du$

Integrate: Use "e" Rule $\int e^x dx \ = e^x + C$

= $\frac{1}{6}e^u + C$

Replace the u with $4x^3$ *and write answer*

Answer: $\frac{1}{6}e^{4x^3} + C$

Example: Rewrite a problem that has a fraction using $u - substitution$, then integrate to find the answer.

$$\int \frac{16}{4x-3} dx$$

First rewrite the problem so that it has a negative exponent instead of a fraction.

$$\int \frac{16}{4x-3} dx = \int \frac{16}{(4x-3)^1} dx \ = \int (4x-3)^{-1} 16dx$$

$(it\ saves\ a\ step\ to\ write\ the\ parenthesis\ first$)

$let\ u = inside\ of\ the\ parenthesis$

$$u = 4x - 3$$

Rewrite the problem so that the "parenthesis is changed to an "u"

$$\int (4x-3)^{-1} 16dx$$

$$= \int u^{-1} 16dx$$

Next find $\frac{du}{dx}$

$$u = 4x - 3$$

$$\frac{d}{dx} u = \frac{d}{dx} 4x - \frac{d}{dx} 3$$

$$\frac{du}{dx} = 4$$

Multiply by dx to clear the fraction.

$dx\frac{du}{dx} = dx * 4$

$du = 4dx$

$the\ Algebra\ give\ me\ \ du = 4dx$

But I need to replace $16xdx$ $\int u^{-1}16dx$

Multiply by 4

$4du = 4 * 4dx$

$4du = 16dx$

Replace $16dx\ with\ 4du$

$\int u^{-1}16dx = \int u^{-1}4du = \int 4u^{-1}du$

Rewrite: using $\int af(x)dx = a\int f(x)dx$

$= 4\int u^{-1}du$

Integrate using:

ln" Rule: $\left\{\int x^{-1}dx = \ln|x| + C\right.$

$= 4ln|u| + C$

Change the $u\ to\ 4x - 3\ and\ write\ the\ answer$

$Answer$: $4ln|4x - 3| + C$

#1 – 12: Use u-substitution to evaluate the indefinite integrals. (Minimum homework all odds)

1) $\int 2x(x^2+5)^4 dx$

2) $\int 3x^2(x^3-4)^5 dx$

Rewrite the problem so that the parenthesis is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the “parenthesis is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

Next replace to make problem only have u's

Next integrate: use Power Rule:

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\int 3x^2(x^3 - 4)^5 dx = \frac{1}{6}(x^3 - 4)^6 + C$

3) $\int (2x+3)\,(x^2+3x-4)^3 dx$

4) $\int (8x+2)\,(4x^2+2x-7)^5 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the “parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

Next replace to make problem only have u’s

Next integrate: use Power Rule:

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C \quad provided\ n \neq -1$$

Last change u $back\ to\ get\ the\ answer$

$answer$: $\int (8x+2)\,(4x^2+2x-7)^5 dx = \frac{1}{6}(4x^2+2x-7)^6 + C$

5) $\int 2xe^{x^2}dx$

6) $\int 3x^2e^{x^3}dx$

Rewrite the problem so that the “e” is written first:

Next: $let\ u = exponent\ of\ the\ e$

Rewrite the problem so that the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

Next replace to make problem only have u’s

Next integrate: "e" Rule $\int e^x dx \;\; = e^x + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\int 3x^2e^{x^3}dx = e^{x^3} + C$

7) $\int (2x+5)\, e^{x^2+5x} dx$

8) $\int (6x^2+8x)\, e^{2x^3+4x^2+3} dx$

Rewrite the problem so that the “e” is written first:

Next: $let\ u = exponent\ of\ the\ e$

Rewrite the problem so that the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

Next replace to make problem only have u’s

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\int (6x^2+8x)\, e^{2x^3+4x^2+3} dx = e^{2x^3+4x^2+3} + C$

9) $\int \frac{4}{4x-7} dx$

10) $\int \frac{10}{10x+3} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

Next replace to make problem only have u's

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\int \frac{10}{10x+3} dx = ln|10x + 3| + C$

11) $\int \frac{2x+3}{x^2+3x-5} dx$

12) $\int \frac{6x+9}{3x^2+9x-2} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the “parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

Next replace to make problem only have u’s

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Last change $u\ back\ to\ get\ the\ answer$

$answer{:}\ \int \frac{6x+9}{3x^2+9x-2} = ln|3x^2 + 9x - 2| + C$

#13-24: Use u-substitution to evaluate the indefinite integrals.

13) $\int 6x(x^2+5)^4 dx$

14) $\int 15x^2(x^3-4)^5 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match

Next replace to make problem only have u's

Next integrate: Power Rule:

Last change $u\ back\ to\ get\ the\ answer$

Answer: $\int 15x^2(x^3-4)^5 dx = \frac{5}{6}(x^3-4)^6 + C$

15) $\int (4x+6)\,(x^2+3x-4)^3 dx$

16) $\int (32x+8)\,(4x^2+2x-7)^5 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match

Next replace to make problem only have u's

Last change $u\ back\ to\ get\ the\ answer$

Answer: $\int (32x+8)\,(4x^2+2x-7)^5 dx = \frac{2}{3}(4x^2+2x-7)^6 + C$

17) $\int 10xe^{x^2}dx$

18) $\int 15x^2e^{x^3}dx$

Rewrite the problem so that the "e" is written first:

Next: $let\ u = exponent\ of\ the\ e$

Rewrite the problem so that the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match.

Next replace to make problem only have u's

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\int 15x^2e^{x^3}dx = 5e^{x^3} + C$

19) $\int(8x + 20)\, e^{x^2+5x} dx$

20) $\int(18x^2 + 24x)\, e^{2x^3+4x^2+3} dx$

Rewrite the problem so that the “e” is written first:

Next: $let\ u = exponent\ of\ the\ e$

Rewrite the problem so that the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match.

Next replace to make problem only have u’s

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer\text{:}\ \int(18x^2 + 24x)\, e^{2x^3+4x^2+3} dx = 3e^{2x^3+4x^2+3} + C$

21) $\int \frac{8}{4x-7}dx$

22) $\int \frac{30}{10x+3}dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the “parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match.

Next replace to make problem only have u's

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\int \frac{30}{10x+3} dx = 3ln|10x + 3| + C$

23) $\int \frac{16x+24}{x^2+3x-5}$

24) $\int \frac{42x+63}{3x^2+9x-2}$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match.

Next replace to make problem only have u's

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Last change $u\ back\ to\ get\ the\ answer$

$answer: \int \frac{42x+63}{3x^2+9x-2} = 7ln|3x^2 + 9x - 2| + C$

#25-32: Use u-substitution to evaluate the indefinite integrals.

25) $\int 2x(4x^2+5)^4 dx$

26) $\int 3x^2(5x^3-4)^5 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match

Next replace to make problem only have u's

Next integrate: Power Rule:

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\int 3x^2(5x^3-4)^5 dx = \frac{1}{30}(5x^3-4)^6 + C$

27) $\int (2x+3)\,(2x^2+6x-1)^3 dx$

28) $\int (4x+1)\,(4x^2+2x-7)^5 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match

Next replace to make problem only have u's

Next integrate: Power Rule:

Last change $u\ back\ to\ get\ the\ answer$

$answer\ \ \int (4x+1)\,(4x^2+2x-7)^5 dx = \frac{1}{12}(4x^2+2x-7)^6 + C$

29) $\int \frac{2}{4x-7}\,dx$

30) $\int \frac{5}{10x+3}\,dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match.

Next replace to make problem only have u's

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Last change $u\ back\ to\ get\ the\ answer$

$answer{:}\ \int \frac{5}{10x+3} dx = \frac{1}{2} ln|10x + 3| + C$

31) $\int \frac{2x+2}{3x^2+6x-5} dx$

32) $\int \frac{2x+3}{3x^2+9x-2} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

Rewrite the problem so that the parenthesis with the exponent is first:

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the “parenthesis with the exponent is changed to an "u"

Next find $\frac{du}{dx}$

Multiply by dx to clear the fraction.

This is not good enough. Multiply to make a perfect match.

Next replace to make problem only have u’s

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

Last change $u\ back\ to\ get\ the\ answer$

$answer\text{:}\ \int \frac{2x+3}{3x^2+9x-2} = \frac{1}{3} ln|3x^2 + 9x - 2| + C$